# Correspondence

## Comments on "Fourier Analysis and Signal Processing by Use of the Möbius Inversion Formula"

V. K. Ananthashayana

*Abstract*—**A new method of Fourier analysis based on the number-theoretic Möbius inversion formula was recently developed by I. S. Reed *et al*. This correspondence suggests a few corrections needed in its proof of theorems and equations.**

In the above paper,[1] on page 460, for the proof of Theorem 5, case II, the initial subcase $b$ should read subcase $a$ with the following modifications:

*Subcase a:* $m$ is an even integer, i.e., $m = 2q$, where $q \in I$, so that $n = 2^k(2(2q) + 1)$.

On page 461, below (22), "$n = 1, 2, \cdots, 10$" should read "$n = 1, 2, \cdots, 5$." In the last paragraph of page 461, "$\overline{A}(n/m)$ in (20)" should read "$\overline{A}(m/n)$ in (22)."

The Fig. 3 caption should be "the magnitude $|H(nf_0)|$ for $-5 < n < 5$ of function $H(nf_0)$" with the plot $|H(nf_0)|$ versus $n$ instead of $H(nf_0)$ versus $n$.

On page 462 under case II while computing $b_2$, $S(2, 1/8) = [\overline{A}(1/8) + \overline{A}(3/8)]/2$ should read $S(2, 1/8) = [\overline{A}(1/8) + \overline{A}(5/8)]/2$.

The proof of Theorem 6 on page 469 after interchanging integration and summation should read

$$F(t) * G(t) = \sum_{n=-N}^{N} g_n e^{i2\pi f_0 n t}\left(\frac{1}{T}\int_0^T e^{-i2\pi f_0 n u} F(u)\, du\right)$$

instead of

$$F(t) * G(t) = \sum_{n=-N}^{N} e^{i2\pi f_0 n t}\left(\frac{1}{T}\int_0^T e^{-i2\pi f_0 n u} F(u)\, du\right).$$

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[1] I. S. Reed, D. W. Tufts, X. Yu, T. K. Truong, M.-T. Shih, and X. Yin, *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 38, no. 3, pp. 458–470, Mar. 1990.

## Minimum-Variance Deconvolution and Maximum-Likelihood Deconvolution for Nonwhite Bernoulli-Gaussian Processes with a Joseph Spectrum

Chong-Yung Chi

*Abstract*—**Todoeschuck and Jensen recently reported that the reflectivity sequences, denoted $\mu(k)$, calculated from some sonic logs are not white and have a power spectral density approximately proportional to frequency, called a Joseph spectrum. In this correspondence, we show how to compute the minimum-variance estimate $\hat{\mu}_{MV}(k)$ and maximum-likelihood estimate $\hat{\mu}_{ML}(k)$ for a $\mu(k)$ modeled as a nonwhite Bernoulli-Gaussian (B-G) process with a Joseph spectrum. We also present the corresponding $\hat{\mu}^*_{MV}(k)$ and $\hat{\mu}^*_{ML}(k)$ for a statistically equivalent white B-G process $\mu^*(k)$ which mimics $\mu(k)$. Through some simulations, we conclude that $\hat{\mu}^*_{MV}(k) = \hat{\mu}_{MV}(k)$ and $\hat{\mu}^*_{ML}(k) = \hat{\mu}_{ML}(k)$ for a white B-G process $\mu(k)$ and that $\hat{\mu}^*_{MV}(k)$ and $\hat{\mu}^*_{ML}(k)$ are acceptable for the estimation of $\mu(k)$ when $\mu(k)$ is nonwhite with a Joseph spectrum.**

### I. Introduction

The estimation of the desired signal $\mu(k)$ from noisy measurements $z(k)$, $k = 1, 2, \cdots, N$, where

$$z(k) = \mu(k) * v(k) + n(k) \tag{1}$$

where $v(k)$ is the impulse response of a linear time-invariant system and $n(k)$ is the measurement noise, is a deconvolution problem. This problem can be found in areas such as seismic deconvolution, biomedical ultrasonic imaging, and channel equalization (communications). Conventionally, the whiteness assumption about $\mu(k)$ is used in seismic deconvolution, such as predictive deconvolution [2], [3], minimum-variance deconvolution (MVD) [4], [5], and maximum-likelihood deconvolution (MLD) [6], [7].

Although the conventional whiteness assumption about $\mu(k)$ is used in seismic deconvolution, perhaps it is valid that $\mu(k)$'s are white for some geologies but not for all geologies. Todoeschuck and Jensen [1] recently reported that the reflectivity sequences calculated from some sonic logs are not white and have a power spectral density approximately proportional to frequency, called a Joseph spectrum. The associated normalized autocorrelation $c(k)$ of $\mu(k)$ defined as

$$c(k) = \frac{E[\mu(i)\mu(i+k)]}{E[\mu^2(i)]} \tag{2}$$

is negligible for $|k| \geq 2$, except $c(1) = c(-1) \approx -0.405$. We wonder whether or not the existing MVD and MLD algorithms, which are based on the whiteness assumption about $\mu(k)$, are still applicable for estimating this nonwhite $\mu(k)$ because this nonwhite model is valid for some geologies according to [1]. This motivated

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a study of the performance of both the well-known MVD and MLD algorithms for this nonwhite $\mu(k)$.

Kormylo and Mendel [6], [7] developed MLD by modeling $\mu(k)$ as a white Bernoulli–Gaussian (B-G) [6], [7] process, denoted $\xi(k)$. This process has been popularly used in modeling a reflection sequence in seismic deconvolution and biomedical ultrasonic imaging. A white B-G process is defined as

$$\xi(k) = r(k) \cdot q(k) \tag{3}$$

where $r(k)$ is a white Gaussian random process with zero mean and variance $C$ and $q(k)$ is an independent Bernoulli process for which

$$P_r[q(k)] = \lambda^{q(k)}(1 - \lambda)^{(1 - q(k))} \tag{4}$$

where $q(k)$ can take only a binary value one or zero and $0 \le \lambda \le 1$ is the probability for $q(k)$ equal to 1. Notice that $\xi(k)$ is a zero-mean white process whose variance is $\sigma_\xi^2(k) = \lambda C$ when $q(k)$ is not known and $\sigma_\xi^2(k) = Cq(k)$ (time-varying variance) when $q(k)$ is known.

In this correspondence, we assume that $n(k)$ is white Gaussian with variance $R$ and that the parameters $C$, $\lambda$, $R$, and the wavelet $v(k)$ are given *a priori*. We show how to obtain the minimum-variance estimate $\hat{\mu}_{MV}(k)$ and the maximum-likelihood estimate $\hat{\mu}_{ML}(k)$ for a nonwhite B-G $\mu(k)$ with a Joseph spectrum, denoted $\hat{\mu}_{MV}(k)$ and $\hat{\mu}_{ML}(k)$, respectively. Next, we form a statistically equivalent white B-G process $\mu^*(k)$ to mimic $\mu(k)$. We obtain the resulting estimates of $\mu^*(k)$, denoted $\hat{\mu}^*_{MV}(k)$ and $\hat{\mu}^*_{ML}(k)$. Through some simulations, we compare $\hat{\mu}_{MV}(k)$ with $\hat{\mu}^*_{MV}(k)$ and $\hat{\mu}_{ML}(k)$ with $\hat{\mu}^*_{ML}(k)$, respectively. We can then infer whether or not both $\hat{\mu}^*_{MV}(k)$ and $\hat{\mu}^*_{ML}(k)$ are acceptable for the estimation of $\mu(k)$.

In Section II, we briefly review the MVD and MLD for a white B-G process. We then present a nonwhite B-G model for $\mu(k)$ with a Joseph spectrum in Section III. In Section IV, we show how to obtain $\hat{\mu}_{MV}(k)$, $\hat{\mu}_{ML}(k)$, $\hat{\mu}^*_{MV}(k)$, and $\hat{\mu}^*_{ML}(k)$ using the existing algorithms. We then show some simulation results in Section V. Finally, we summarize our conclusions.

## II. MVD and MLD for a White B-G Process

In this section, we briefly review MVD and MLD for a white B-G $\mu(k) = \xi(k)$ defined as (3), respectively. We now turn to MVD.

### A. MVD

It is well known that the linear minimum-variance estimate, $\hat{\boldsymbol{\mu}}_{MV}$, of $\boldsymbol{\mu}$ is given by

$$\hat{\boldsymbol{\mu}}_{MV} = E[\boldsymbol{\mu}\boldsymbol{z}]\{E[\boldsymbol{z}\boldsymbol{z}']\}^{-1}\boldsymbol{z} \tag{5}$$

where $\boldsymbol{\mu} = (\mu(1), \mu(2), \cdots, \mu(N))'$ and $\boldsymbol{z} = (z(1), z(2), \cdots, z(N))'$. $\mu(k)$ is thought of as a zero-mean white random process with variance $\sigma_\mu^2(k) = \lambda C$ in the derivation of $\hat{\boldsymbol{\mu}}_{MV}$. Therefore, $\hat{\boldsymbol{\mu}}_{MV}$ is a function of $\lambda C$, $R(k) = R$ (the variance of $n(k)$), wavelet $v(k)$, and measurement $z(k)$. Instead of directly computing $\hat{\boldsymbol{\mu}}_{MV}$ using (5), Mendel [4], [5] developed a computationally efficient MVD filter. The estimate $\hat{\mu}_{MV}(k)$ is computed via a Kalman filter type optimal smoother associated with a standard state-variable model as follows:

$$\boldsymbol{x}(k) = \Phi \boldsymbol{x}(k-1) + \gamma\mu(k) \tag{6}$$

$$z(k) = \boldsymbol{h}'\boldsymbol{x}(k) + n(k) \tag{7}$$

where $\Phi$ is an $m \times m$ matrix, $\gamma$ and $\boldsymbol{h}$ are $m \times 1$ vectors, and $m$ is the system order. Of course, there are many $(\Phi, \gamma, \boldsymbol{h})$'s which generate the same output $z(k)$ (e.g., [7], [8]). Note that $v(k) = \boldsymbol{h}'\Phi^k\gamma$ for $k \ge 0$. We remark that the MVD filter is applicable for either time-varying or nonstationary systems or both.

### B. MLD

Kormylo and Mendel [6] and Mendel [7] developed a maximum-likelihood deconvolution (MLD) algorithm which includes the detection of $q(k)$ and estimation of $r(k)$ instead of directly estimating $\mu(k)$ as in MVD. The maximum-likelihood estimate, $\hat{\boldsymbol{q}}_{ML}$, of $\boldsymbol{q} = (q(1), q(2), \cdots, q(N))'$ is the one that maximizes the likelihood function

$$S\{\boldsymbol{q}|z\} = p(z, \boldsymbol{q}). \tag{8}$$

After the detection of $q(k)$ is completed, the maximum-likelihood estimate $\hat{\boldsymbol{r}}_{ML}$, of $\boldsymbol{r} = (r(1), r(2), \cdots, r(N))'$ is then computed. $\hat{\boldsymbol{r}}_{ML}$ is the one that maximizes the likelihood function

$$S'\{\boldsymbol{r}|z\} = p(z, \boldsymbol{r}|\boldsymbol{q}). \tag{9}$$

Since $\boldsymbol{z}$ and $\boldsymbol{r}$ are jointly Gaussian when $\boldsymbol{q}$ is given

$$\hat{\boldsymbol{r}}_{ML} = \hat{\boldsymbol{r}}_{MV} = E[\boldsymbol{r}\boldsymbol{z}]\{E[\boldsymbol{z}\boldsymbol{z}']\}^{-1}\boldsymbol{z}. \tag{10}$$

Therefore, $\hat{\boldsymbol{r}}_{ML}$ can also be obtained using Mendel's MVD filter with $\sigma_\mu^2(k) = C\hat{q}_{ML}(k)$ (time-varying variance). Finally, the maximum-likelihood estimate of $\mu(k)$ is given by

$$\hat{\mu}_{ML}(k) = \hat{r}_{ML}(k) \cdot \hat{q}_{ML}(k). \tag{11}$$

Chi *et al.* [9] further developed a computationally fast MLD algorithm which is practical and has been successfully used to process real seismic data. Let us emphasize that the detection of $q(k)$ and estimation of $r(k)$ in these MLD algorithms require the standard state-variable model (6) and (7). How the MVD and MLD algorithms provide the estimates of $\mu(k)$ is omitted here. The reader can refer to [4], [5] for the former and [6], [7], [9] for the latter.

## III. Bernoulli–Gaussian Process with a Joseph Spectrum

Let $\mu(k)$ be a nonwhite B-G process with a Joseph spectrum. We prefer to use $\mu(k)$ for the signal to be estimated without confusion, although it denotes a different random process from that used in Section II.

As described in Section I, for a $\mu(k)$ with a Joseph spectrum, its normalized autocorrelation function $c(k)$ is negligible for $|k| \ge 2$. We, therefore, assume that

$$c(k) = 1 + \alpha\,\delta(k-1) + \alpha\,\delta(k+1) \tag{12}$$

where $\delta(k)$ is the discrete delta function ($\delta(k) = 1$, for $k = 0$ and $\delta(k) = 0$ for $k \ne 0$). We model $\mu(k)$ such that its normalized autocorrelation function is given by (12) as follows:

$$\mu(k) = \xi(k) + \rho\xi(k-1) \tag{13}$$

where $\xi(k)$ is a white B-G process defined as (3). It can easily be shown that $\mu(k)$ has a normalized autocorrelation function given by (12) and

$$\alpha = \frac{\rho}{1 + \rho^2}. \tag{14}$$

Note that $\mu(k) = \xi(k)$ when $\rho = 0$ and that $\rho$ can easily be obtained by solving (14) when $\alpha$ is given. Next, we discuss how we estimate $\mu(k)$.

## IV. Estimation of a B-G Process with a Joseph Spectrum

From (13) one can see that the minimum-variance estimate $\hat{\mu}_{MV}(k)$ and the maximum-likelihood estimate $\hat{\mu}_{ML}(k)$ can be obtained, respectively, as follows:

$$\hat{\mu}_{MV}(k) = \hat{\xi}_{MV}(k) + \rho\hat{\xi}_{MV}(k-1) \tag{15}$$

and

$$\hat{\mu}_{ML}(k) = \hat{\xi}_{ML}(k) + \rho\hat{\xi}_{ML}(k-1). \tag{16}$$

From (1) and (13) we see that

$$z(k) = \xi(k) * \{1, \rho\} * v(k) + n(k). \tag{17}$$

Because $\xi(k)$ is a white B-G process, the existing MVD and MLD algorithms described in Section II are applicable for estimating $\xi(k)$ if we can convert (17) into a standard state-variable form, as in (6) and (7). Thus, what we need to do for computing both $\hat{\xi}_{MV}(k)$ and $\hat{\xi}_{ML}(k)$ is to form a standard state-variable model with $\xi(k)$ being the input.

Let

$$x_1(k) = \xi(k). \tag{18}$$

Substituting (13) and (18) into (6) and (7) provides the following augmented state-variable equations:

$$\begin{bmatrix} x_1(k) \\ \boldsymbol{x}(k) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \rho\gamma & \Phi \end{bmatrix} \begin{bmatrix} x_1(k-1) \\ \boldsymbol{x}(k-1) \end{bmatrix} + \begin{bmatrix} 1 \\ \gamma \end{bmatrix} \xi(k) \tag{19}$$

and

$$z(k) = [0 \;\; \boldsymbol{h}'] \begin{bmatrix} x_1(k) \\ \boldsymbol{x}(k) \end{bmatrix} + n(k) \tag{20}$$

which is exactly a standard state-variable model required by the MVD and MLD algorithms described in Section II. After we obtain $\hat{\xi}_{MV}(k)$ and $\hat{\xi}_{ML}(k)$, $\hat{\mu}_{MV}(k)$ and $\hat{\mu}_{ML}(k)$ can be obtained using (15) and (16), respectively.

We now derive a statistically equivalent white B-G process, denoted, $\mu^*(k)$ by letting $\Pr[\mu^*(k) \neq 0] = \Pr[\mu(k) \neq 0]$ and $E[(\mu^*(k))^2] = E[\mu^2(k)]$. Let

$$\mu^*(k) = r^*(k) \cdot q^*(k). \tag{21}$$

Then, the parameter $\lambda^*$, the probability of $q^*(k) = 1$, is determined by

$$\begin{aligned} \lambda^* &= \Pr[q^*(k) = 1] = \Pr[\mu^*(k) \neq 0] = \Pr[\mu(k) \neq 0] \\ &= \Pr[\xi(k) \neq 0 \text{ or } \xi(k-1) \neq 0] \\ &= \Pr[q(k) \neq 0 \text{ or } q(k-1) \neq 0] = 1 - (1-\lambda)^2 \cong 2\lambda \end{aligned} \tag{22}$$

for a small $\lambda$. Because $C = E[\xi^2(k)]/\lambda$, the amplitude variance, $C^*$, of $r^*(k)$ is determined by

$$C^* = \frac{E[(\mu^*(k))^2]}{\lambda^*} = \frac{E[(\mu^2(k)]}{\lambda^*} = \frac{1+\rho^2}{2} C. \tag{23}$$

We can then compute $\hat{\mu}^*_{MV}(k)$ and $\hat{\mu}^*_{ML}(k)$ with the statistical parameters $\lambda^*$ and $C^*$ using the algorithms described in Section II. Notice that when $\rho = 0$, (22) and (23) are not applicable, and $\hat{\mu}^*_{MV}(k) = \hat{\mu}_{MV}(k)$ and $\hat{\mu}^*_{ML}(k) = \hat{\mu}_{ML}(k)$ since $\mu^*(k) = \mu(k) = \xi(k)$. Next, we show some simulation results for $\hat{\mu}_{MV}(k)$, $\hat{\mu}_{ML}(k)$, $\hat{\mu}^*_{MV}(k)$, and $\hat{\mu}^*_{ML}(k)$.

## V. Simulation Results

In this section, we present some simulation results using synthetic data. The noise-free data were generated by first convolving $\{1, \rho\}$ with a Bernoulli–Gaussian sequence $\xi(k)$ with parameters $\lambda = 0.07$ and $C = 0.0225$ to form the nonwhite $\mu(k)$, and then convolving $\mu(k)$ with a selected wavelet $v(k)$. We then added pseudo-Gaussian random noises to the noise-free data to form the synthetic noisy data. We then computed $\hat{\xi}_{MV}(k)$, as well as $\hat{\mu}^*_{MV}(k)$ using Mendel's MVD filter and $\hat{\xi}_{ML}(k)$ as well as $\hat{\mu}^*_{ML}(k)$ using the Kormylo and Mendel's MLD algorithm. Finally, we obtained $\hat{\mu}_{MV}(k)$ and $\hat{\mu}_{ML}(k)$ using (15) and (16), respectively.

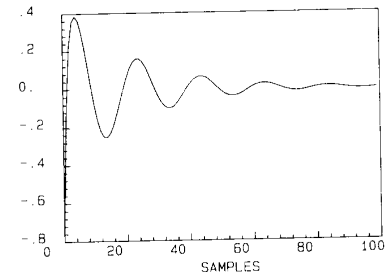


Fig. 1. Wavelet $v(k)$.

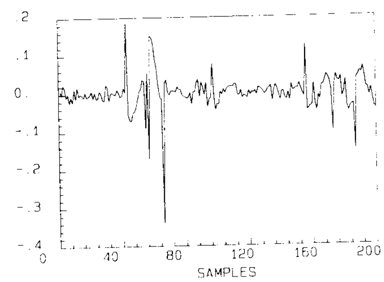


Fig. 2. Synthetic noisy data with SNR = 10.

The selected wavelet $v(k)$, which was taken from [7], is shown in Fig. 1. The selected parameter $\alpha$ (see (12)) was equal to $-0.405$ as reported in [1]. The parameter $\rho$ was then equal to $-0.51$ (see (14)). The synthetic noisy data is shown in Fig. 2 with signal-to-noise ratio (SNR) equal to 10. The estimates $\hat{\mu}_{MV}(k)$ and $\hat{\mu}_{ML}(k)$ are shown in Figs. 3(a) and (b), respectively. The estimates $\hat{\mu}^*_{MV}(k)$ and $\hat{\mu}^*_{ML}(k)$ are shown in Figs. 4(a) and (b), respectively. In these figures, *'s denote true spikes and bars denote estimated spikes.

Comparing Fig. 3(a) with Fig. 4(a), one can see that the results shown in Fig. 3(a) are only slightly better than those shown in Fig. 4(a). Comparing Fig. 3(b) with Fig. 4(b), again, one can see that the results shown in Fig. 3(b) are only slightly better than those shown in Fig. 4(b). From these simulation results, we infer that the performances of the MVD filter and the MLD algorithm for the presented statistically equivalent white B-G process are very close to those of the corresponding optimal estimates. In other words, $\hat{\mu}^*_{MV}(k)$ and $\hat{\mu}^*_{ML}(k)$ are acceptable for the estimation of $\mu(k)$ when $\mu(k)$ is nonwhite with a Joseph spectrum.

## VI. Conclusions

In seismic deconvolution, the reflection sequence $\mu(k)$ is conventionally assumed to be white. A white B-G model for a reflectivity sequence may be valid for some geologies, and, a nonwhite B-G model with a Joseph spectrum may be valid for other geologies.

In this correspondence, we have shown how to obtain both the minimum-variance and maximum-likelihood estimates for a nonwhite B-G process $\mu(k)$ with a Joseph spectrum. This nonwhite $\mu(k)$ is viewed as a colored noise generated as the output of a (2-point

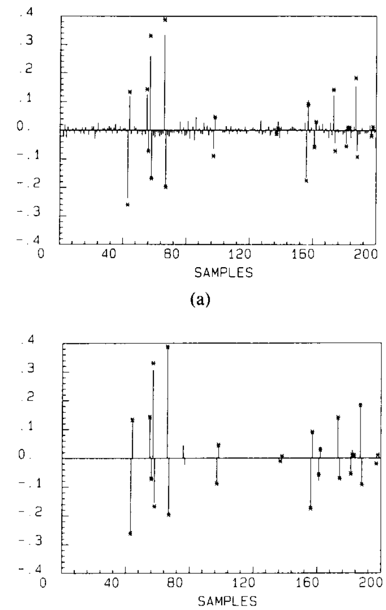


Fig. 3. (a) Minimum-variance estimate $\hat{\mu}_{MV}(k)$ and (b) maximum-likelihood estimate $\hat{\mu}_{ML}(k)$. *'s denote true spikes and bars denote estimates.

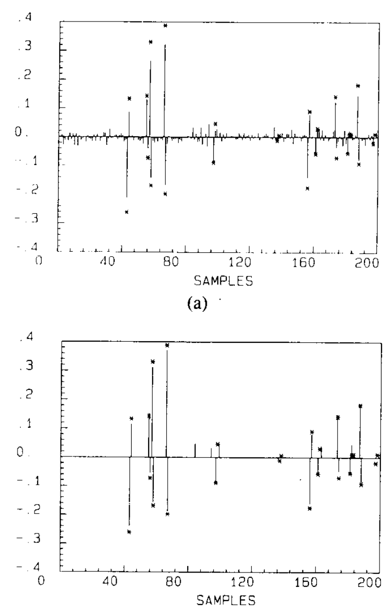


Fig. 4. (a) Minimum-variance estimate $\hat{\mu}^*_{MV}(k)$ and (b) maximum-likelihood estimate $\hat{\mu}^*_{ML}(k)$. *'s denote true spikes and bars denote estimates.

FIR) coloring filter that is excited by white (B-G) noise $\xi(k)$. Thus, the results presented in this correspondence are a special case of this more general concept. We also have presented the corresponding estimates associated with a statistically equivalent white B-G process $\mu^*(k)$ which mimics $\mu(k)$. The simulation results showed that the estimates, $\hat{\mu}^*_{MV}(k)$ and $\hat{\mu}^*_{ML}(k)$, of $\mu^*(k)$ are very close to the optimal estimates $\hat{\mu}_{MV}(k)$ and $\hat{\mu}_{ML}(k)$, of $\mu(k)$, respectively. We, therefore, conclude that $\hat{\mu}^*_{MV}(k) = \hat{\mu}_{MV}(k)$ and $\hat{\mu}^*_{ML}(k) = \hat{\mu}_{ML}(k)$ for a white B-G $\mu(k)$ and that $\hat{\mu}^*_{MV}(k)$ and $\hat{\mu}^*_{ML}(k)$ are acceptable for the estimation of $\mu(k)$ when $\mu(k)$ is nonwhite with a Joseph spectrum.

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### References

[1] J. P. Todoeschuck and O. G. Jensen, "Joseph geology and seismic deconvolution," *Geophysics*, vol. 53, no. 11, pp. 1410–1414, 1988.
[2] E. A. Robinson, "Predictive decomposition of seismic traces," *Geophysics*, vol. 22, pp. 767–778, 1957.
[3] E. A. Robinson and S. Treitel, *Geophysical Signal Analysis*. Englewood Cliffs, NJ: Prentice-Hall, 1980.
[4] J. M. Mendel, "White noise estimators for seismic data processing in oil exploration," *IEEE Trans. Automat. Contr.*, vol. AC-22, pp. 694–706, 1977.
[5] J. M. Mendel, "Minimum-variance deconvolution," *IEEE Trans. Geosci. Remote Sensing*, vol. GE-19, no. 3, pp. 161–171, July 1981.
[6] J. Kormylo and J. M. Mendel, "Maximum-likelihood deconvolution," *IEEE Trans. Geosci. Remote Sensing*, vol. GE-21, pp. 72–82, 1983.
[7] J. M. Mendel, *Optimal Seismic Deconvolution: An Estimation-Based Approach*. New York: Academic, 1983.
[8] S. Y. Kung, "A new identification and model reduction algorithm via singular value decompositions," presented at the 12th Asilomar Conf. Circuits, Syst., Comput., Pacific Grove, CA, 1978.
[9] C.-Y. Chi, J. M. Mendel, and D. Hampson, "A computationally fast approach to maximum-likelihood deconvolution," *Geophysics*, vol. 49, pp. 550–565, May 1984.

# On the Quality of Recursively Identified FIR Models

Svante Gunnarsson

***Abstract*—In this correspondence we consider recursive identification of time-varying systems having finite impulse response, focusing on the tradeoff between tracking capability and disturbance rejection. Approximate, but simple and explicit, frequency domain expressions for the model quality are derived for three different identification algorithms. The results, derived under the assumptions of slow adaptation, slow system variation, and high model order, are extensions of the results presented in [1] to the case where the system output is affected by correlated disturbances.**

## I. Problem Description

A fundamental problem in recursive identification of signals and systems having time varying properties is the tradeoff between tracking ability and disturbance rejection. This problem has been discussed by several authors, and surveys are given in, for example, [2] and [3]. We shall in this correspondence consider the prob-

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